

Lattice Approach to Diquark Condensation in Dense Matter

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Results are presented of a Monte Carlo simulation of a three dimensional Gross-Neveu model with $SU(2) \otimes SU(2)$ chiral symmetry at non-zero baryon chemical potential μ , corresponding to non-zero baryon density. The phenomenon of quark pair condensation is sought via measurement of a two point function. For μ sufficiently large there is a sharp transition between a phase where the chiral symmetry is broken by a condensate $\langle \bar{q}q \rangle$ and one where a scalar diquark condensate $|\langle qq \rangle| \neq 0$. Global $U(1)$ baryon number symmetry may remain unbroken, however, due to the absence of long range order in the phase of $\langle qq \rangle$.

Recent work [1,2] suggests that new forms of ordering may exist at high baryon density. In addition to the standard $\langle \bar{q}q \rangle$ condensate associated with spontaneous breaking of chiral symmetry it has been proposed that the existence of a Fermi surface combined with any attractive qq interaction results in the formation of a $\langle qq \rangle$ condensate of quark Cooper pairs. Remarkably, this *diquark condensate* is associated with baryon number violation. Formation of a diquark condensate, which is non-invariant under the vectorlike $U(1)$ global symmetry of baryon number is generally forbidden by the Vafa-Witten (VW) theorem [3]. Notable exceptions to the VW theorem include theories with complex measures and models with Yukawa couplings to scalar degrees of freedom, such as the coupling to σ in (1) below.

For $\mu \neq 0$ the fermion determinant of QCD is complex and lattice simulations fail dramatically however the lattice approach *can* be successfully applied at $\mu \neq 0$ to toy field theories. I will present work done in collaboration with S.J. Hands which is the first lattice study of diquark condensation at $\mu \neq 0$ implemented in the Gross-Neveu (GN) model with $SU(2) \otimes SU(2)$ chiral symmetry and formulated in $d = 2 + 1$ space-time dimensions. With the exception of confinement this model incorporates all of the essential physics.

The lattice model we have simulated has the

following Euclidean path integral:

$$Z = \int D\sigma D\vec{\pi} \det(M^\dagger M[\sigma, \vec{\pi}]) e^{-\frac{2}{g^2}(\sigma^2 + \vec{\pi} \cdot \vec{\pi})} \quad (1)$$

where M is the staggered fermion matrix; σ and the triplet $\vec{\pi}$ are real auxiliary fields defined on the dual sites \tilde{x} of a three dimensional Euclidean lattice. The lattice model is formulated such that the integration measure is real and positive. The full lattice action (involving isodoublet staggered lattice fermion fields χ and ζ) and its symmetries are fully discussed in [4]. The corresponding $d = 3 + 1$ model is outlined in [5].

In the limit $m \rightarrow 0$, our model is invariant under a global symmetry akin to the $SU(2)_L \otimes SU(2)_R$ of the continuum NJL model. For $T = \mu = 0$, a mean field treatment shows that this $SU(2) \otimes SU(2)$ symmetry is spontaneously broken to $SU(2)_{isospin}$ by the generation of a condensate $\Sigma_0 = \langle \sigma \rangle = \frac{2}{g^2} \langle \bar{\chi} \chi \rangle$ and the fermion acquires physical mass $m_f = \Sigma_0$. The symmetry is broken for coupling $g^2 > g_c^2 \simeq 1.0$. The numerical results presented here are for a value $g^2 = 2.0$, corresponding to a theory deep in the broken phase for $\mu = 0.0$, and with $\Sigma_0 = 0.706(1)$ in units of inverse lattice spacing, ie. rather far from the continuum limit.

For $T, \mu \neq 0$ a mean field solution is also known [6], in which $\Sigma(\mu, T)$ is expressed in terms of Σ_0 . At $T = 0$ the basic feature is that Σ remains constant as μ is increased up to a critical value $\mu_c = \Sigma_0$, whereupon Σ falls sharply to zero (in the

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chiral limit), signalling a first order chiral symmetry restoring transition. Monte Carlo simulations [6] support this picture, even when massless Goldstone excitations (with the quantum numbers of the π field) are present [7].

We performed simulations of the model (1) on $L_s^2 \times L_t$ lattices with bare mass $m = 0.01$ at a coupling $1/g^2 = 0.5$. The simulation method is a standard hybrid Monte Carlo algorithm. We monitored the chiral condensate $\langle \bar{\chi}\chi \rangle$, and the baryon density n using stochastic estimators.

We have constructed two local diquark operators which are anti-symmetric in spatial, flavour and isospin indices. Firstly the *spectral scalar*: $\chi^p(x)\tau_2^{pq}\chi^q(x)$ which is even under parity and secondly the *spectral pseudoscalar*: $(-)^{x_1+x_2+x_3}\chi^p(x)\tau_2^{pq}\chi^q(x)$ which is odd under parity.

Measurement of diquark condensates on the lattice demands innovative techniques and for this initial study we chose an indirect method: the two point function $\langle qq(0)\bar{q}\bar{q}(x) \rangle$ was measured as the expectation of the product of two fermion propagators as in QCD baryon spectroscopy, and the condensate extracted by assuming clustering at large spacetime separation, ie.

$$\langle qq(0)\bar{q}\bar{q}(x) \rangle = \langle qq(0)\bar{q}\bar{q}(x) \rangle_c + \langle qq \rangle \langle \bar{q}\bar{q} \rangle, \quad (2)$$

the latter term on the right being proportional to $|\langle qq \rangle|^2$. A non-zero condensate signal therefore shows up as a plateau in the timeslice correlator $G(t)$ at large time separation, and may be extracted by a fit of the form

$$\begin{aligned} G(t) &= \sum_{x_1, x_2} \langle qq(0)\bar{q}\bar{q}(t, x_1, x_2) \rangle \\ &= A \exp(-M_+ t) + B \exp(-M_- (L_t - t)) \\ &\quad + C^2(L_s) |\langle qq \rangle|^2, \end{aligned} \quad (3)$$

with M_{\pm} the masses of forward and backward propagating diquark states respectively. The value of the constant $C(L_s)$ is not determined *a priori*. Naively one expects $C = L_s$, and hence the plateau height to be an extensive quantity. The trend in the diquark condensates as we pass from broken to symmetric phase is clear from Fig. 1. There is a critical value, $\mu_c \simeq 0.65$, at which the chiral $\langle \bar{q}q \rangle$ condensate falls sharply

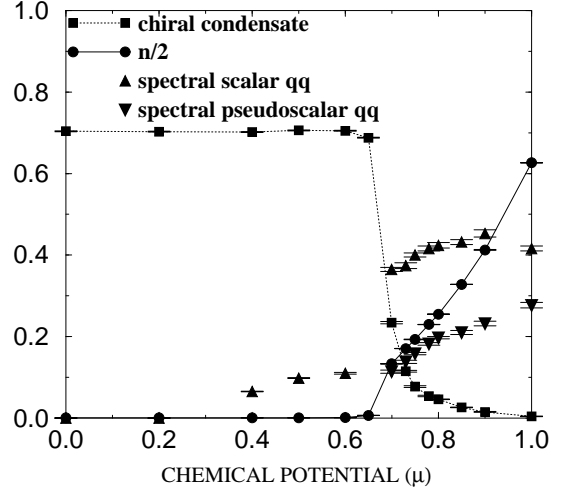


Figure 1. Overview of observables $\langle \bar{\chi}\chi \rangle$, n and diquark condensate signals $C|\langle qq \rangle|$ as functions of μ , obtained on a $16^2 \times 24$ lattice, showing the onset of diquark condensation at $\mu_c \simeq 0.65$.

and the number density (n) begins to rise from zero. The spectral scalar $C|\langle qq \rangle|$ condensate rises slowly from zero for $0.4 \leq \mu < \mu_c$, jumps upwards discontinuously close to μ_c , and continues to rise steadily. For $\mu > 1.0$ as the number density approached saturation the scalar qq propagators became distorted. The spectral pseudoscalar $C|\langle qq \rangle|$ signal was consistent with zero for $\mu < \mu_c$ and no fits to the propagators were possible but for $\mu > \mu_c$ this condensate is non-zero and increases with μ . It is considerably smaller in magnitude than the spectral scalar. This pseudoscalar condensate is *parity violating*, and therefore it would be remarkable if the signal in the chirally symmetric phase were to remain non-zero in the large volume limit.

The trend in the M_{\pm} data [4] was clear with small masses in the symmetric phase and large masses in the broken phase. The forward propagating M_+ states were more difficult to extract from the fits than the backward propagating M_- states. For $\mu < \mu_c$, M_- decreased linearly with μ reflecting the trend observed in the physical fermion mass m_f in previous simulations [7]. M_- was approximately constant for $\mu > \mu_c$. The zero density fermion mass $m_f = \Sigma_0 \simeq 0.7$, defines the ratio of physical to cutoff scales and the observed

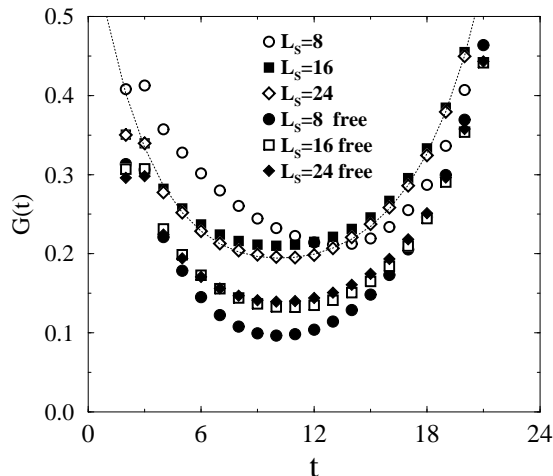


Figure 2. Spectral scalar diquark correlator at $\mu = 0.8$ for both free and interacting quarks on $L_s^2 \times 24$ lattices.

qq states had masses around 0.3 which is comparatively light.

Fits to the spectral scalar correlator in the symmetric phase ($\mu = 0.8$) were very stable as L_t was increased from 16 to 40. The smaller parity violating signal, on the other hand, decreased with increasing L_t . The non-zero spectral scalar condensate in the broken phase is at least in part due to finite lattice size. To determine the behaviour with spatial volume we next performed a series of runs on $L_s^2 \times 24$ lattices at $\mu = 0.8$, with L_s varying from 8 to 24. To our surprise we found very little change as L_s increased, the fitted value of $C|\langle qq \rangle|$ more or less saturating for $L_s \geq 16$. Results for $L_s = 8, 16, 24$ are shown in Fig. 2, together with the fit for $L_s = 24$. For comparison we also plot the equivalent correlators for free fermions at $\mu = 0.8$; it is striking that for this case the trend as L_s increases is in the opposite direction. We conclude that the constant C is roughly independent of L_s .

The fact that the two-point correlation function exhibits clustering at large temporal separation is suggestive of diquark condensation at $\mu \neq 0$. In order to quantify the measurement, however, it will be necessary to provide a numerical estimate for the constant C in (3).

The spontaneous breaking of the $U(1)$ sym-

metry usually implies the existence of a Goldstone mode associated with long wavelength fluctuations in the phase of the $\langle qq \rangle$ condensate. However, in the absence of an explicit diquark source (analogous to a bare mass for the case of $\langle \bar{q}q \rangle$), one would expect large finite volume effects, generically proportional to $L^{-(d'-2)}$, where d' is the dimension of the effective field theory describing the fluctuations of the order parameter [8]. We therefore speculate that the effective field theory describing the spatial correlations of the diquark correlator has $d' = d - 1 = 2$, and that there is no long range order in the spatial directions [9]. A similar distinction between temporal and spatial correlations has been observed in instanton liquid models [2]. This would account for the volume-independence of C . Strictly speaking, therefore, we have not observed a true condensation. This interpretation of our results can be tested by simulations of the equivalent $3 + 1$ dimensional model [5], where we *would* expect long range order. An extension of this work will involve the introduction of a diquark source term to study the one point function. This will serve to calibrate our two point function measurement and to extend our study to other diquark operators and the $T \neq 0$ regime.

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